Dipole Excitations in 2D insulators
Quantum Lévy flights

Boris Altshuler
Physics Department, Columbia University

Collaborations: Igor Aleiner, Kostya Efetov,

The March MIFP Meeting 2011

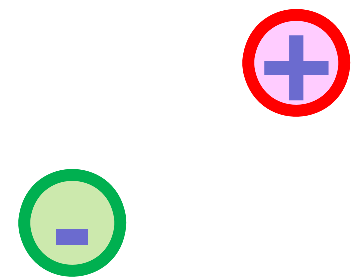
March 16-20, 2011 - Marino (Roma), Italy

Mediterranean Institute of Fundamental Physics

Part 1.

Weak localization of neutral dipole excitations

Neutral Dipole Excitations

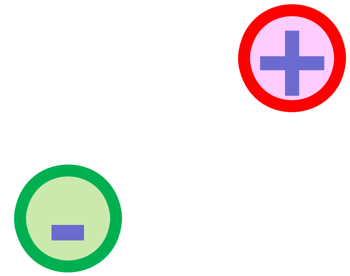


- excitons in semiconductors,
- optical phonons in the polar crystals,
- dipole excitations in granular superconductors,
- vacancy-interstitial excitations in Wigner crystals
- vortex – antivertex pairs
- . . . .

Manifest themselves:

- Thermal transport
- Heat bath for charges
- Interaction with photons

Neutral Dipole Excitations



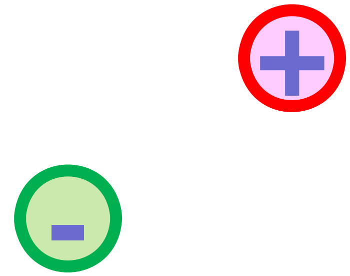
- excitons in semiconductors,
- optical phonons in the polar crystals,
- dipole excitations in granular superconductors,
- vacancy-interstitial excitations in Wigner crystals
- vortex – antivertex pairs
- . . . .

dipole moment - 2D vector $\vec{d}$

2D dipole-dipole interaction:

$$U_{\vec{d}_1,\vec{d}_2}(\vec{r}) = \frac{|\vec{r}|^2\, \vec{d}_1 \bullet \vec{d}_2 - 2(\vec{d}_1 \bullet \vec{r})(\vec{d}_1 \bullet \vec{r})}{|\vec{r}|^4}$$

Neutral Dipole Excitations



dipole moment - 2D vector $\vec{d}$

2D dipole-dipole interaction:

$$U_{\vec{d}_1,\vec{d}_2}\left(\vec{r}\right) = \frac{\left|\vec{r}\right|^2 \vec{d}_1 \bullet \vec{d}_2 - 2\left(\vec{d}_1 \bullet \vec{r}\right)\left(\vec{d}_1 \bullet \vec{r}\right)}{\left|\vec{r}\right|^4}$$

Corresponds to the logarithmic interaction between the charges

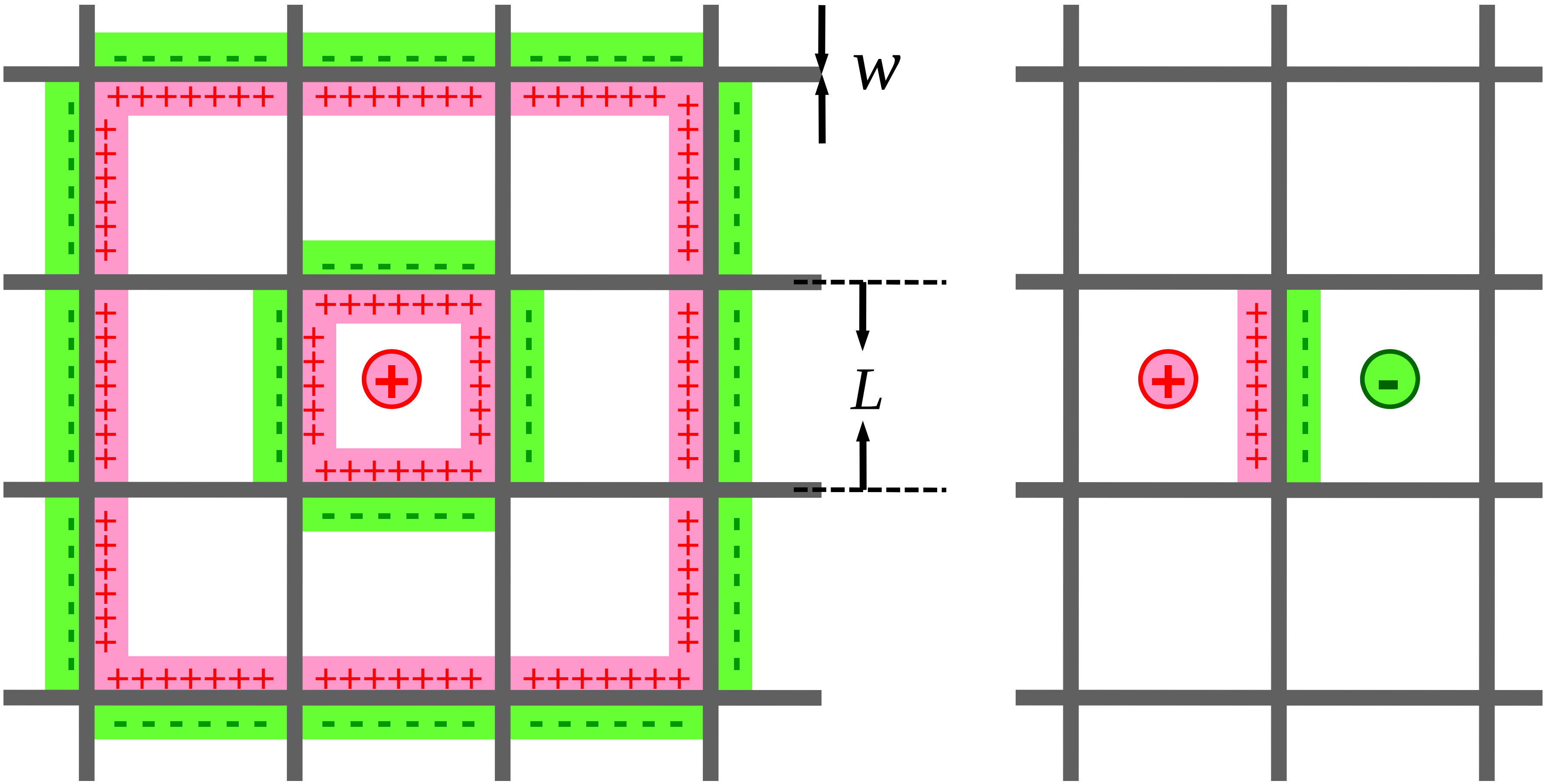
Correct at intermediate distances:

- **Coulomb interaction – provided that the dielectric constant of the film is large**
- **Vertex-antivertex pair – at distances smaller than the magnetic length**

One boson - charge

Fazio & Schon, (1991).

Two bosons - dipole



$$E_c = \frac{2e^2}{C_0}$$

Energies

$$E_d = \frac{2e^2}{C}$$

$$L >> w \quad \Rightarrow \quad C_0 << C \quad \Rightarrow \quad E_c >> E_d$$

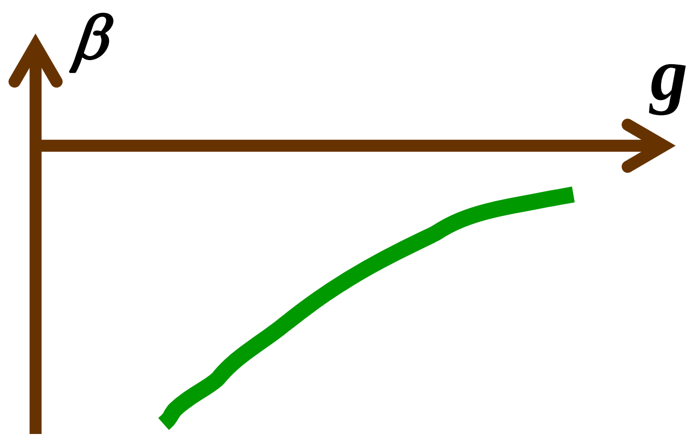
Quenched disorder. 2D Anderson localization.

- One particle in a random potential
- Mean free path, mean free time
- Diffusion, diffusion constant D
- Conductivity σ = conductance $G(L)$; L is the system linear size– classical picture
- Dimensionless Thouless conductance

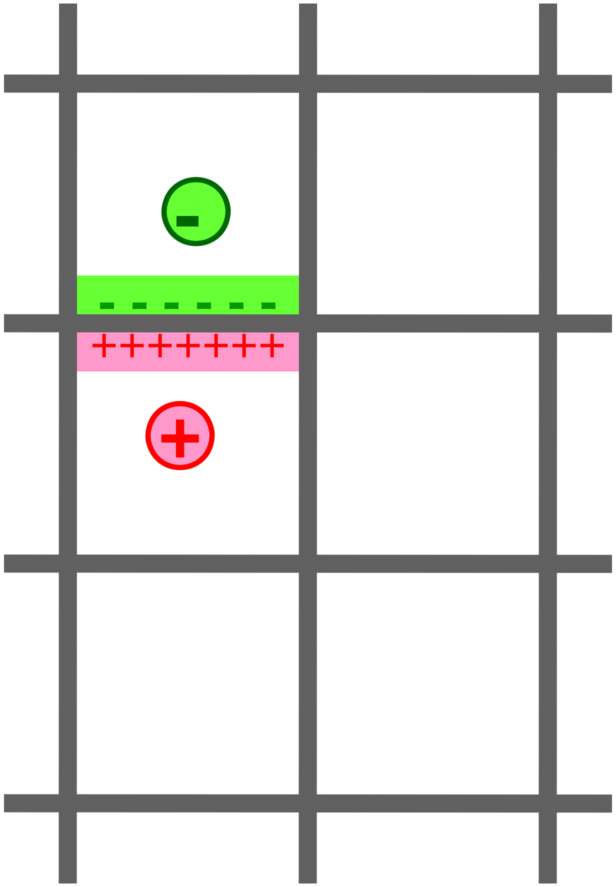
$$g \equiv \frac{e^2}{\hbar} G = \nu D$$

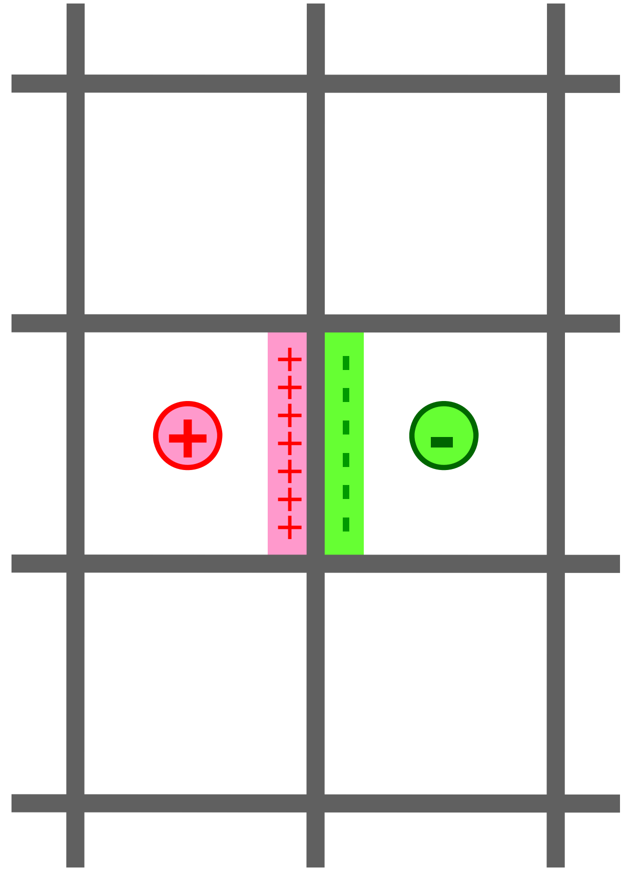
- Quantum Anderson Localization $g = g(L) \xrightarrow[L\to\infty]{} 0$
- Scaling theory:

$$\frac{dg}{d(\log L)} = \beta(g) \xrightarrow[g\to\infty]{} -\frac{1}{2\pi^2} + O\left(\frac{1}{g}\right)$$



Back to dipoles. Long range hops





- Dipole-dipole interaction – exchange by a virtual photon
- Emission – absorption of the photon – long range hop

$$\hat{H}_{lr} = \lambda \int d\vec{r}_1 d\vec{r}_2 \sum_{\alpha,\beta=x,y} \hat{b}^+_\beta(\vec{r}_2)\hat{b}_\alpha(\vec{r}_1)\frac{|\vec{r}_{12}|^2\delta_{\alpha\beta} - 2(r_{12})_\alpha (r_{12})_\beta}{|\vec{r}_{12}|^4}$$

Long range hops

$$\hat{H}_{lr} = \lambda \int d\vec{r}_1 d\vec{r}_2 \sum_{\alpha,\beta=x,y} \hat{b}^{+}_{\beta}(\vec{r}_2)\hat{b}_{\alpha}(\vec{r}_1)\frac{|\vec{r}_{12}|^2 \delta_{\alpha\beta} - 2(r_{12})_{\alpha}(r_{12})_{\beta}}{|\vec{r}_{12}|^4}$$

⇓

Logarithmic corrections to the Thouless conductance, which compete with the weak localization contribution

$$\delta g_{lr}(L) = +\frac{A}{2\pi^2}\delta(\log L)$$

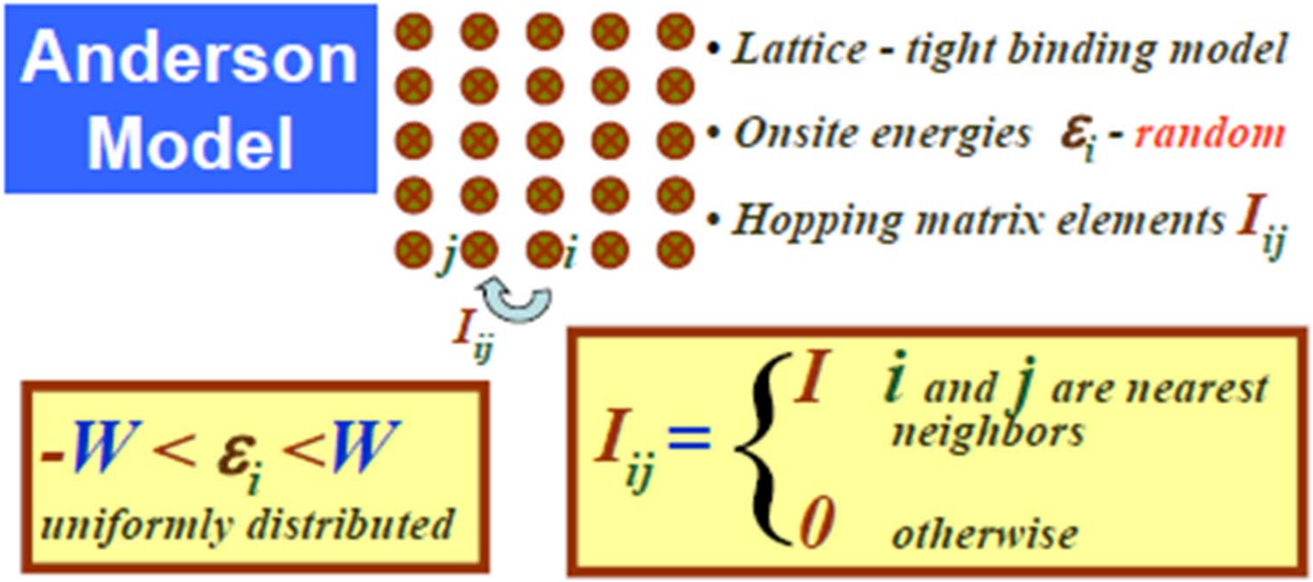
$$0 \le A \le 1$$

Important:

Although we are dealing with long range effects one can introduce additional fields and describe these effects in a framework of a local theory. This distinguishes the dipole hops from generic long range hops.

Reason: Poisson equation is differential rather that integral

Consequence: the theory is renormalizable



What if $I_{ij} \propto \dfrac{1}{\left|\vec{r}_i - \vec{r}_j\right|^2}$ **?**

$$\delta g_{lr}(L) = +\frac{A}{2\pi^2}\log L \qquad 0 \le A \le 1$$

Renormalizable theory ⟹ RG equations

I. T-invariant systems – orthogonal ensemble:

$$\frac{\partial g(L)}{\partial \log L} = +\frac{A-1}{2\pi^2} + \frac{A}{8\pi^2 g} + O\left(\frac{1}{g^2}\right) \qquad g >> 1$$

Note that

- the critical point is at large g if A is close to **1**
- this critical point is **stable**
- In order to determine this point at a given A one has to extend calculations to the second loop

I. T-invariant systems – orthogonal ensemble:

$$\frac{\partial g(L)}{\partial \log L} = +\frac{A-1}{2\pi^2} + \frac{A}{8\pi^2 g} + O\left(\frac{1}{g^2}\right) \qquad g >> 1$$

Note that

- the critical point is at large g if A is close to **1**
- this critical point is **stable**
- In order to determine this point at a given A one has to extend calculations to the second loop

I. Broken T-invariance – unitary ensemble:

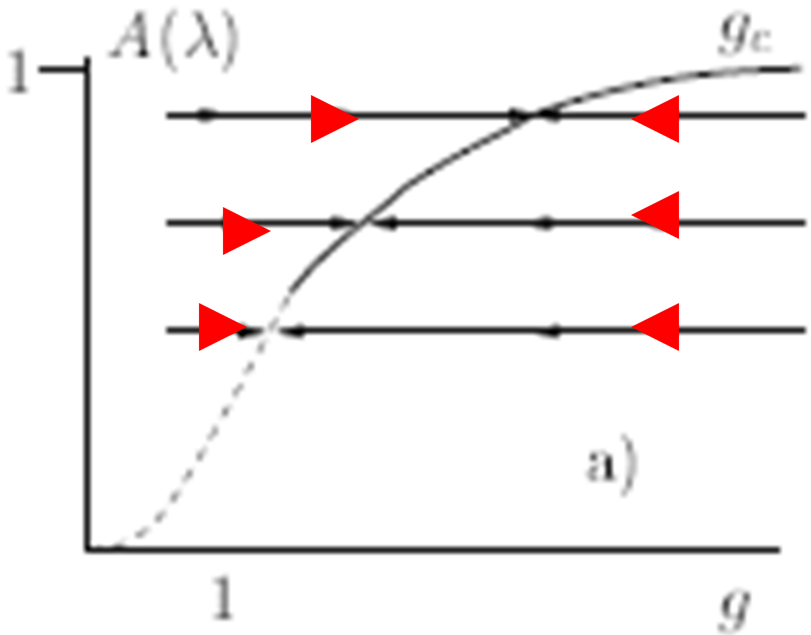
$$\frac{\partial g(L)}{\partial \log L} = +\frac{A}{2\pi^2} - \frac{1}{8\pi^4 g} + O\left(\frac{1}{g^2}\right) \qquad g >> 1$$

Note that

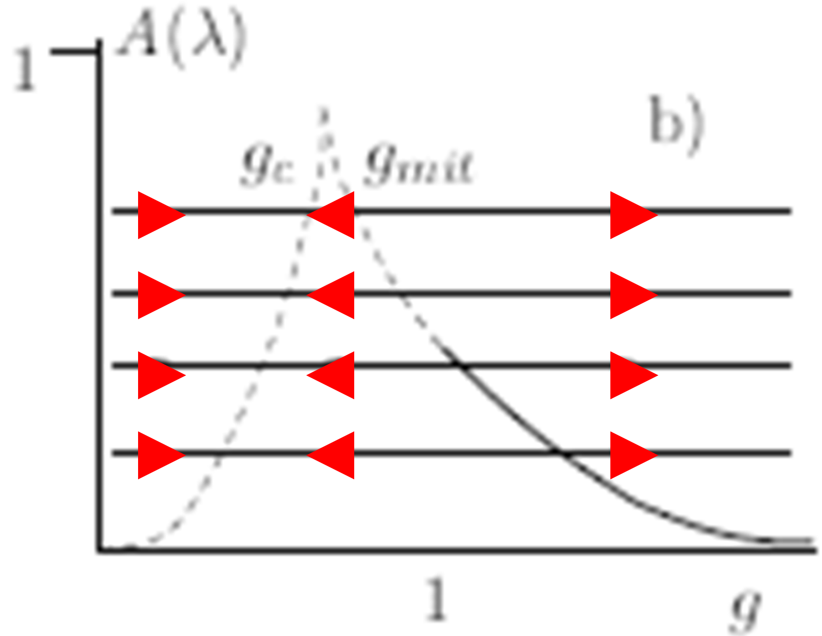
the critical point is at large g if A is small

this critical point is **unstable** – metal - ? transition

RG flows



orthogonal



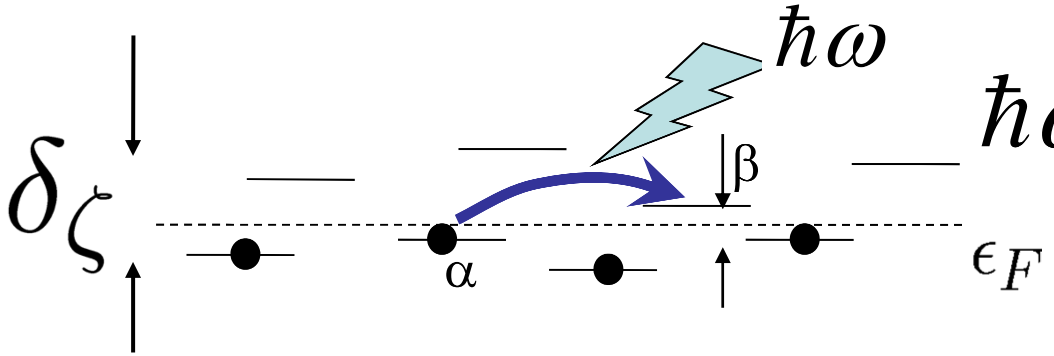
unitary

g=0 is an unstable point ⟹ Dipoles are never truly localized

Part 2.

transport in insulators

Phonon-assisted variable range hopping



$$\hbar\omega = \varepsilon_\alpha - \varepsilon_\beta$$

Variable Range Hopping

N.F. Mott (1968)

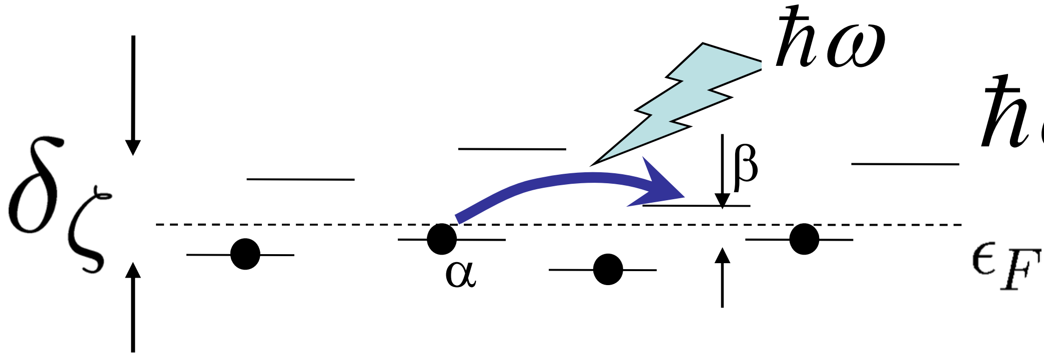
$$\sigma(T) \propto T^{\gamma} \exp\left[-\left(\frac{\delta_\zeta}{T}\right)^{\frac{1}{d+1}}\right]$$

Mechanism-dependent prefactor

Optimized phase volume

Any bath with a continuous spectrum of delocalized excitations down to $\boldsymbol{\omega = 0}$ will give the same exponential

Phonon-assisted variable range hopping



$$\hbar\omega = \varepsilon_\alpha - \varepsilon_\beta$$

Variable Range Hopping

N.F. Mott (1968)

$$\sigma(T) \propto T^\gamma \, \mathrm{exp}\left[-\left(\frac{\delta_\zeta}{T}\right)^{\frac{1}{d+1}}\right]$$

Stretched exponent:

$$R(T) \propto \exp[(T_0/T)^\alpha]$$

$$\alpha = \frac{1}{d+1}$$ without e-e interactions — **Mott**

$$\alpha = \frac{1}{2}$$ long range Coulomb e-e interactions — **Efros-Shklovskii**

$$\alpha < 1$$

Experimental facts (puzzles)

1. Arrhenius law in the temperature dependence of the resistance

$$R(T) \propto \exp(T_0/T)$$

D. Kowal & Z. Ovadyahu, 2007

2. Prefactors are close to $\hbar/e^2$ and do not contain high powers of temperature following from the phonon density of states, etc.

Nonlinear I-V characteristics Ovadia, Sacepe´ and Shahar 2009

Theory BA, Kravtsov, Lerner and Aleiner 2009

3. Why heating?

4. In "normal" systems (SC suppressed by magnetic field) the energy relaxation rate is the same in metals and in insulators $\propto T^6$, while in SC systems it is exponential

Speculations

Kostya Efetov, Sergey Syrzanov, Igor Aleiner, and BA
unpublished

Model: Josphson array with $J < E_c \ll \Delta$
weak disorder.

Q: What are charge carrying excitations?

A: Cooper pairs – bosons with charge $2e$

Q: Why Ahrenius?

A: Spectrum of the bosons has a gap $\sim E_c$.
As a result boson concentration is exponentially small
DoS at the chemical potential is zero

Q: Why it is not always like this?

A: Lifshits tails

Speculations 2

Q: Why Ahrenius is more common for SC than for normal metal **?**

A: Lifshits tails are thicker for single charge than for double charge particles.

Q: What is the mechanism of the charge transport **?**

A: Bosons with exponentially small concentration are delocalized in the many-body sence if T is not too low.

It is most probable that dipoles are delocalized. Charges may be localized if left alone. However delocalized dipoles can cause hoping conductivity in the same way phonons were supposed to do. Temperature dependence of the conductivity would be determined by the concentration of charges. Temperature dependence of the energy relaxation rate should be also exponential – concentration of dipoles

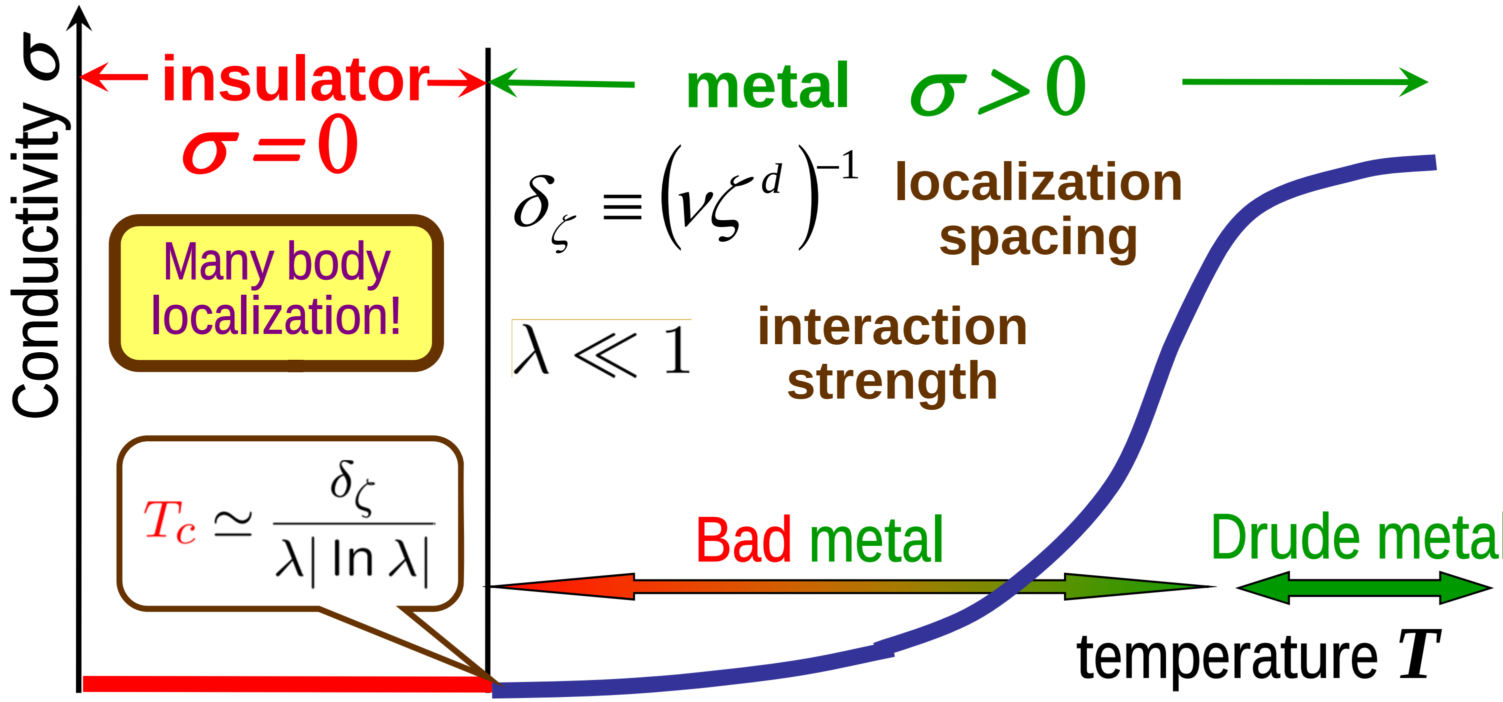
Speculations 3

It is most probable that dipoles are delocalized. Charges may be localized if left alone. However delocalized dipoles can cause hoping conductivity in the same way phonons were supposed to do. Temperature dependence of the conductivity would be determined by the concentration of charges. Temperature dependence of the energy relaxation rate should be also exponential – concentration of dipoles

Q: What is the role of the phonons?

A: No role at all in ohmic regime.
At finite voltage, when heating takes place, it is interaction of the dipoles with phonons that determines effective temperature.

Many-body localization of interacting electrons



No many-body localization as long as the dipoles can be viewed as 2D