Thermodynamic model of the macroscopically ordered exciton state

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Outline

- Introduction: Butov experiment
- Macroscopically ordered exciton state (MOES)
- Transport theory of the MOES
- Thermodynamic model
- Comparison with the experiment
Indirect excitons in CQW’s

The exciton lifetime is long due to the spatial separation between e and h

Spatial PL patterns in photoexcited CQW structure.

Low temperatures.

The external ring is a source of cold excitons

Å Theory of the MOES based on the kinetics of exciton formation at low T has been proposed:


\[
\begin{align*}
\partial_t n_e &= D_e \nabla^2 n_e - wn_e n_h - J_e, \\
\partial_t n_h &= D_h \nabla^2 n_h - wn_e n_h + J_h, \\
\partial_t n_x &= D_x \nabla^2 n_x + wn_e n_h - \gamma n_x \\
\end{align*}
\]

\[
w \sim f = e^u, \quad u \equiv \frac{n_x}{n_0(T)}, \quad n_0(T) = \frac{2gm_x k_B T}{\pi \hbar^2} \\
f = 1 + N_E^{eq}
\]

Å Stimulated scattering in real space?
Å The model doesn’t take into account the dipolar exciton repulsion
Experiments carried out after the publication of the Levitov paper:

Å Repulsive interaction in the MOES:

Å Extended spontaneous coherence in the MOES:

OES as a thermodynamic system...
Thermodynamic model of the MOES

Each bead is considered as a coherent aggregate having macroscopical spin $S$

\[ \Psi(x, y) = \sqrt{\frac{N_0}{\pi}} \frac{1}{r} \exp\left(-\frac{x^2 + y^2}{2r^2}\right) \]

\[ N_0 = \frac{N}{n} \]

$N_0$ spins form a macroscopical spin $S$

The whole ring is a canonical ensemble

\[ F = -k_B T \ln Z \quad Z = z^n \]

$z$ - partition function for one bead

$n$ - the number of the beads
Calculation of the partition function for one bead

Microstate of a bead: $S_j = \begin{pmatrix} N_0 - j \\ j \end{pmatrix}$

Corresponding interaction energy:

$$E_j = \frac{V_0 N_0^2}{4\pi r^2} - \frac{\alpha}{\pi r^2} j(N_0 - j)$$

Free energy of n beads:

$$F(n, r) = \frac{(V_0 - \alpha) N^2}{4\pi r^2 n} + \frac{nk_B T}{2} \ln\left(\frac{\alpha}{\pi r^2 k_B T}\right)$$
Spin-spin interaction of the nearest neighbours

\[ J = u \int \Psi_1(\vec{r}) \Psi_2(\vec{r}) d\vec{r} \]

\[ E_{ss} = J \langle S^2 \rangle \]

Modified free energy of the system:

\[ F(x, n) = \frac{(V_0 - \alpha)N^2}{4\pi^3 R^2} x^{-2} n + N \cdot k_B T \frac{u}{\alpha} R^2 x^2 e^{-x^2} n^{-2} \]

\[ x = \frac{2r}{d} \]

\[ \langle S^2 \rangle = \frac{\sum_{j=0}^{N_0} S_j^2 \exp(-E_j / kT)}{z} \]
minimum of the free energy

\[ F(x, n) = \frac{(V_0 - \alpha)N^2}{4\pi^3 R^2} x^{-2} n + N \cdot k_B T \frac{u}{\alpha} R^2 x^2 e^{-x^2} n^{-2} \]

\[
\begin{aligned}
\frac{\partial F}{\partial x} &= 0 \\
\frac{\partial F}{\partial n} &= 0 \\
\left\{ \frac{\partial F}{\partial x} \right\} &= n = 2\pi \left( \frac{R}{\rho} \right)^{4/3} \\
\rho &= \left( \frac{Ne\alpha(V_0 - \alpha)}{u \cdot k_B T} \right)^{1/4} \\
x &= 1
\end{aligned}
\]

The bead diameter:

\[ x = \frac{nr}{\pi R} \implies 2r = \frac{x \cdot \rho^{4/3}}{R^{1/3}} \]
Comparison with the experiment: the number of the beads

\[ n = 2\pi \left( \frac{R - R_0}{\rho} \right)^{4/3} \]

- **Excitation power is fixed**
  - T=1.4 K
  - Parameters:
    - \( \rho = 14 \, \mu m \)
    - \( R_0 = 50 \, \mu m \)

- **Gate voltage is fixed**
  - T=1.4 K
  - Parameters:
    - \( \rho = 12 \, \mu m \)
    - \( R_0 = 51 \, \mu m \)
Comparison with the experiment: 
the bead diameter

$$2r = \frac{x \cdot \rho^{4/3}}{(R - R_0)^{1/3}}$$

**Graph 1:**
- Title: Excitation power is fixed
- Description: $T = 1.4$ K
- Parameters:
  - $R = 103 \, \mu m$
  - $\rho = 14 \, \mu m$
  - $x = 1.72$

**Graph 2:**
- Title: The ratio $x = 2r/d$ doesn't change with the radius
Conclusions

Å The thermodynamic approach to the phenomena of the MOES is formulated
Å We minimize the free energy and obtain the dependences of:
  1. The number of the beads on the ring radius
  2. The size of a bead on the ring radius
Å The theory reproduces the main experimental features
Thanks for your attention!
Other parameters estimation:

Using experimental value of the blue-shift $\sim 1 \text{ meV}$

and the formula

$$F(n, r) = \frac{(V_0 - \alpha) N^2}{4\pi^2 n}$$

one can estimate

$$(V_0 - \alpha) \sim 10 \text{ mkeV} \cdot \text{mkm}^2$$

$$V_0 = \frac{de^2}{\varepsilon \varepsilon_0} \approx 17 \text{ } \mu\text{eV} \cdot \text{mkm}^2 \quad \alpha \approx 6.5 \text{ } \mu\text{eV} \cdot \text{mkm}^2$$

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$$\rho = \left( \frac{N e \alpha (V_0 - \alpha)}{u \cdot k_B T} \right)^{1/4} \quad N \sim 10^6 \quad u = 150 \text{ mkeV}$$